Almost Global Robust Attitude Tracking Control of Spacecraft in Gravity

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In this paper, we treat the practical problem of tracking the attitude and angular velocity of a spacecraft in the presence of gravity and disturbance moments. Autonomous trajectory tracking is a practical problem for robotic spacecraft, as well as autonomous aerial and ground vehicles. The approach used here achieves near global stable trajectory tracking by using a globally defined dynamics model that includes a moment on the vehicle created by a gravity potential and a disturbance moment that vanishes when the required angular velocity to be tracked is zero. The feedback control law is also globally defined. In the presence of the particular type of disturbance moments considered, this control law achieves almost global asymptotic tracking. Lyapunov methods are used to analyze the properties of the closed loop system and show near global stability and asymptotic convergence to the desired attitude and angular velocity trajectory. The treatment in this paper utilizes concepts from geometric mechanics to treat the dynamics of the feedback system in a global manner.

I. Introduction

Control of the attitude dynamics of a rigid body in the presence of gravity and intermittent bounded disturbance moments is an important and practical control problem with several applications, like aerospace and underwater vehicles, robotic ground vehicles, and spacecraft. Attitude control of a rigid body has been a benchmark problem in nonlinear control. This problem has been studied under various assumptions and scenarios; a sample of the literature in this area can be found in.1–3,5,7,17,18 As shown by Crouch and Bonnard,4 and Crouch,5 no globally asymptotically stabilizing control law that is also continuously dependent on attitude in SO(3) exists.

Most of the prior research on attitude control and stabilization of a rigid body has been carried out using minimal three-coordinate or quaternion representations of the attitude. Any minimal representation necessitates a local analysis since such representations have kinematic singularities and cannot represent attitude globally. The quaternion representation is ambiguous, since for every possible attitude there are two sets of unit quaternion representations. In particular, unit quaternion representations cannot distinguish between attitudes that differ by a 360° rotation about a principal axis. This leads to an undesirable situation where after a rigid body reaches a desired attitude, an arbitrary small perturbation may rotate it by 360° about some axis.4,5 This is similar to the “unwinding phenomenon” as reported by Bhat and Bernstein in trying to obtain global asymptotic stability of attitude motion using minimal attitude representations.2

The control schemes using minimal coordinates or quaternions can globally asymptotically stabilize motion in R³ or S³ respectively using continuous feedback control. However, global asymptotic stability in R³ or S³ does not imply global asymptotic stability in SO(3). Indeed, globally asymptotically stabilizing control laws in that were continuous in quaternions and angular velocities but discontinuous in attitude were reported by Wen.17 These control laws using quaternion and angular velocity feedback could globally asymptotically stabilize motion in S³ × R³ but not in SO(3) × R³ ≃ TSO(3). Since discontinuous control laws are difficult to implement in practice for attitude control using most actuators like reaction wheels or magnetic torquers, we treat the global stability and tracking problem using continuous attitude and angular velocity feedback and a global attitude representation in SO(3). Some recent advances on attitude stabilization in gravity using a
global analysis and continuous state feedback have been reported. Based on these results, we expand our
treatment to obtain a control scheme that results in almost global asymptotic tracking of a given attitude
and angular velocity time trajectory.

This global analysis is applied to the task of asymptotically tracking a desired attitude and angular
velocity profile. This is an important but difficult problem, due to the nonlinear nature of the attitude
dynamics, the presence of uncertainties in the model, and the difficulty of obtaining a global or almost global
tracking control scheme. Prior research using the $H_\infty$ control method for nonlinear systems carry out
a local nonlinear analysis to achieve robustness properties like disturbance attenuation. This paper is an
effort at filling this gap in the research on rigid body attitude control by formulating a theoretical control
scheme for the global tracking control problem. As remarked earlier, this problem is important because of
its several practical applications, like the attitude and angular velocity control of a spacecraft with gravity,
atmospheric and solar forces and moments.

This paper extends the work on attitude stabilization by Chaturvedi and McClamroch to attitude
and angular velocity tracking in the presence of gravity and bounded external disturbance moments that
vanish under certain conditions on the attitude dynamics. The geometric treatment in this work makes it
convenient to deal with the global attitude dynamics and almost global trajectory tracking. A brief outline
of this paper is given here. In Section II, the model of the attitude dynamics is provided, and the attitude
and angular velocity tracking problem is formulated. A tracking control law which asymptotically tracks
the desired trajectory almost globally is presented in Section III. The almost global asymptotic properties
of this control law are shown analytically through results presented in Section IV. Simulation results for
an earth-orbiting satellite in circular orbit in Section V demonstrate the applicability of this attitude and
angular velocity tracking control algorithm in the presence of a bounded disturbance torque. The concluding
Section VI has a summary of results presented, and presents possible future developments.

II. Attitude Dynamics Model and Problem Formulation

The attitude of a rigid body is the relative orientation of a body-fixed coordinate frame to an inertial
frame. The coordinate axes of these frames are related by a linear transformation given by a proper orthog-
onal matrix, called the rotation matrix. The rotation matrix can also be represented by coordinate sets, like
the Euler angles, quaternions, or Rodrigues parameters. We represent the attitude globally by the group
of proper orthogonal matrices. These matrices form a group under matrix multiplication, denoted by $SO(3)$,
where

$$SO(3) = \{ R \in \mathbb{R}^{3 \times 3} : R^T R = I = RR^T, \det(R) = 1 \},$$

where $I$ denotes the $3 \times 3$ identity matrix. We denote the attitude of the rigid body by $R \in SO(3)$.

II.A. Equations of Motion for Attitude Dynamics

We consider the attitude dynamics of a rigid body in the presence of a potential that is determined by the
attitude, a disturbance moment about the center of mass of the body, and control moments. Let $\Omega \in \mathbb{R}^3$
be the angular velocity of the body measured in the body-fixed frame. The attitude kinematics equation, which gives the time rate of change of attitude, is

$$\dot{R} = R\Omega^\times,$$

where ($\cdot^\times : \mathbb{R}^3 \rightarrow so(3)$) is the skew-symmetric mapping given by

$$\Omega^\times = \begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{bmatrix}^\times = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix}.$$

We denote the space of $3 \times 3$ real skew-symmetric matrices by $so(3)$; this is the Lie algebra of the Lie group
$SO(3)$. Note that ($\cdot^\times$) is also the cross-product in $\mathbb{R}^3$: $x^\times y = x \times y$.

Let $M_d(t) \in \mathbb{R}^3$ denote a time varying, bounded disturbance moment acting on the rigid body; this may
be caused by aerodynamic or hydrodynamic drag, solar radiation pressure, or some other natural cause. We
assume that the disturbance moment affecting the rigid body in a gravity-like attitude-dependent potential is
bounded with known bounds, and the signs of the moment components are known. Both these assumptions
are realistic and practical, since these moments are usually caused by forces offset from the center of mass
of the body, and the directions and offsets of these forces are reasonably well-known. We assume that the
disturbance input has a known bound on its norm, as follows:
\[ \| M_d \| \leq \kappa < \sigma, \]  
(2)
where \( \kappa \) and \( \sigma \) are known positive constants.

Let \( U : \text{SO}(3) \to \mathbb{R} \) be a gravity potential dependent on the attitude. The attitude dynamics is given by
the following equations of motion
\[ J \dot{\Omega} + \Omega \times J \Omega = M_g(R) + M_d + \tau, \]  
(3)
where \( \tau \) is the control input (torque) vector, and \( M_g(R) \) is the moment due to the gravity potential \( U(R) \):
\[ M^*_g(R) = \left( \frac{\partial U}{\partial R} \right)^\top R - R^T \frac{\partial U}{\partial R}, \quad R = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \in \text{SO}(3), \quad \text{or} \]
\[ M_g(R) = r_1 \times v_{r_1} + r_2 \times v_{r_2} + r_3 \times v_{r_3}, \quad \frac{\partial U}{\partial R} = \begin{bmatrix} v_{r_1} \\ v_{r_2} \\ v_{r_3} \end{bmatrix}. \]
The partial derivative \( \frac{\partial U}{\partial R} \in \mathbb{R}^{3 \times 3} \) is defined such that \( \left( \frac{\partial U}{\partial R} \right)_{ij} = \frac{\partial U}{\partial R_{ij}} \). A similar dynamics model, without
control torques, has been recently used to study the attitude estimation problem.\(^{16}\)

II.B. Trajectory Tracking Problem Formulation

In this part of this section, we introduce the attitude and angular velocity trajectory tracking problem, and
formulate it in terms of tracking errors in attitude and angular velocity. We specify the desired trajectory
by the initial attitude \( R_d(0) \) and the angular velocity as a function of time \( \Omega_d(t) \), for some interval of time
\( t \in [0, T] \), where \( T > 0 \). Further, \( \Omega_d(t) \) and \( \dot{\Omega}_d(t) \) are bounded during this time interval, so that the attitude
rate of change is given by
\[ \dot{R}_d(t) = R_d(t) \Omega_d(t)^\times, \quad \text{given } R_d(0), \Omega_d(t). \]  
(4)
We first define the attitude and angular velocity tracking errors as follows:
\[ Q(t) \triangleq R_d^\top(t)R(t), \quad \omega(t) \triangleq \Omega(t) - \Omega^\top(t)\Omega_d(t). \]  
(5)
These definitions and equation (4) lead to the attitude error kinematics equation:
\[ \dot{Q} = Q(\Omega - \Omega^\top \Omega_d)^\times = Q \omega^\times. \]  
(6)
Note that the attitude error kinematics is also “left-invariant” like the original attitude kinematics (1), i.e.,
if \( Q_l = CQ \) where \( C \in \text{SO}(3) \) is constant, then \( \dot{Q}_l = Q_l \omega^\times \).

Now we re-write the attitude dynamics equation (3) by substituting for \( \Omega(t) = \omega(t) + \Omega^\top(t)\Omega_d(t) \) from
(5). We also evaluate the gravity potential \( U(R) \) as a function of the attitude error \( Q(t) \) after substituting
\( R(t) = R_d(t)Q(t) \) from (5). Therefore, we obtain:
\[ J \dot{\omega} = J(\omega^\times Q^\top \Omega_d - \Omega^\top \dot{\Omega}_d) - (\omega + \Omega^\top \Omega_d)^\times J(\omega + \Omega^\top \Omega_d) + M_g(R_dQ) + M_d + \tau. \]  
(7)
The trajectory tracking error kinematics and dynamics (6) and (7) depend on \( Q, \omega, \Omega_d, \dot{\Omega}_d, \) the moment
due to the potential \( M_g = M_g(R_dQ) \), and the disturbance and control inputs \( M_d \) and \( \tau \).
III. Attitude and Angular Velocity Tracking Control

In this section, we develop a control law that achieves the control task of asymptotically tracking the desired attitude and angular velocity as specified by (4), following the kinematics and dynamics (6)-(7). The controller must also asymptotically track the desired trajectory while rejecting the bounded but persistent disturbance, with bounds as given by (2). The controller obtained achieves almost global asymptotic tracking, i.e., the attitude and angular velocity converge to the desired trajectory from all except a negligible set. By negligible, we mean set that is open and dense in the tangent bundle $TSO(3)$ and is a set of measure zero.

Let $\Phi : \mathbb{R}^+ \to \mathbb{R}^+$ be a $C^2$ function that satisfies $\Phi(0) = 0$ and $\Phi'(x) > 0$ for all $x \in \mathbb{R}^+$. Furthermore, let $\Phi'(\cdot) \leq \alpha(\cdot)$ where $\alpha(\cdot)$ is a Class-$K$ function.\(^{11}\) The control law design is based on Lyapunov analysis.

Let $L > 0$ and $K > 0$ be control gain matrices, with $K = \text{diag}(k_1, k_2, k_3)$ such that $0 < k_1 < k_2 < k_3$. Therefore $\Phi(\text{trace}(K - KQ))$ is a Morse function on $SO(3)$ (where $Q \in SO(3)$) whose critical points are non-degenerate and hence isolated, according to the Morse lemma.\(^{13}\) Along the kinematics (6), the time derivative of this function is:\(^6\)

$$
\frac{d}{dt}\Phi(\text{trace}(K - KQ)) = -\Phi'(\text{trace}(K - KQ))\text{trace}(KQ\omega^\times)
= -\Phi'(\text{trace}(K - KQ))\omega^T [k_1e_1^\times Q^T e_1 + k_2e_2^\times Q^T e_2 + k_3e_3^\times Q^T e_3].
$$

(8)

Define the vector $u \in \mathbb{R}^3$ as

$$
u \triangleq \frac{\omega}{\|\omega\|} \text{ when } \omega \neq 0,
$$

$$
u \triangleq 0 \text{ when } \omega = 0.
$$

(9)

Now applying the Cauchy-Schwartz inequality to $\mathbb{R}^3$, we get:

$$
\omega^T M_d \leq \|\omega\|\|M_d\| < \sigma \|\omega\|
$$

(10)

when $\|\omega\| \neq 0$.

We propose the following control law to asymptotically track the desired attitude and angular velocity:

$$
\tau = -L\omega + JQ^T \hat{\Omega}_d + (Q^T \Omega_d)^\times JQ^T \Omega_d + \Phi'(\text{trace}(K - KQ))[k_1e_1^\times Q^T e_1 + k_2e_2^\times Q^T e_2 + k_3e_3^\times Q^T e_3] - M_g - \sigma u.
$$

(11)

Note that this control law, and hence the trajectories of the closed-loop system, are continuous with respect to the error variables $Q$ and $\omega$, except at the surface $\omega = 0$ where there is a removable discontinuity. This eliminates chattering, which may occur when tracking control laws that are discontinuous across a sliding surface are used. Several techniques exist to avoid problems during numerical computation of the unit vector $u$ as $\omega \to 0$. We claim that $(Q, \omega) = (I, 0)$ is a stable equilibrium of the closed loop system consisting of (6)-(7) and (11). Further, in the absence of any disturbance moment or when the disturbance moment vanishes as $\omega \to 0$, this equilibrium is almost globally asymptotically stable. By “almost globally” we mean that the domain of attraction is the whole state space (which is the tangent bundle $TSO(3)$), except for a negligible set.

IV. Almost Global Asymptotic Trajectory Tracking

The control law presented in the last section can be shown to asymptotically track the desired attitude and angular velocity trajectory, assuming that actuator saturation bounds are not exceeded by the control torque generated using (11). We show the asymptotic properties of the trajectory tracking errors by analyzing the closed-loop system in this section.

IV.A. Asymptotic Convergence Results

Let $\langle \cdot, \cdot \rangle$ denote the trace inner product on the vector space $\mathbb{R}^{n \times n}$, given by

$$
\langle A, B \rangle \triangleq \text{trace}(A^T B).
$$

We first present a couple of lemmas that are used to prove the main result on asymptotic trajectory tracking.
Let the variation on SO(3) be given by \( \partial Q \) consider the the special case where the perturbing moment \( M \) in Sanyal,\(^{15}\) Lemma 1. which is equivalent to \( KQ \)

We get the criticality condition

which leads to the set of four critical points for \( T \) to find the minimum of the function \( \Phi \)

This gives us the unique minimum point of \( \Phi \)

Since for \( Q \in SO(3) \) we require \( QQ^T = I \), we have

Since we also require \( \det(Q) = 1 \) for \( Q \in SO(3) \), the set of solutions \( S \) of \( S \) for the above equation is given by

This leads to the set of four critical points for \( Q = K^{-1}S \) given by equation (12).

To find the minimum of the function \( \Phi \)

we set its second variation with respect to \( Q \) at a critical point to be positive, as follows:

Since \( U^2 \geq 0 \) for \( U \in \mathfrak{so}(3) \), the above condition is equivalent to \( KQ = S \) being positive definite. From the set of critical solutions \( S \) for \( S \), we see that the only positive definite solution is \( S_0 = K = \text{diag}(k_1,k_2,k_3) \).

This gives us the unique minimum point of \( \Phi \)

The lemma above is a special case of similar results obtained for the attitude estimation problem treated in Sanyal,\(^{15}\) where Wahba’s problem in attitude determination is treated using similar techniques. Next, we consider the the special case where the perturbing moment \( M_d \) vanishes when \( \omega \) vanishes, i.e., \( M_d(\omega, t) \to 0 \) as \( \omega \to 0 \). In this case, we can show almost global asymptotic stability of the equilibrium \((I, 0)\). We first prove the local asymptotic stability of the equilibrium \((I, 0)\).

Lemma 2. The equilibrium \((I, 0)\) of the closed-loop system given by (6)-(7) and control law (11) is locally asymptotically stable and the other equilibria given by \((Q_e, 0)\), where \( Q_e \in E_c \setminus \{I\} \) are unstable. Furthermore, the set of all initial conditions converging to the equilibrium \((Q_e, 0)\), where \( Q_e \in E_c \setminus \{I\} \) form a lower dimensional manifold.

Proof. We use the exponential coordinates \( \Theta \in \mathbb{R}^3 \) given by \( Q = \exp(\Theta^*) \) to describe the attitude tracking error here. Let \( X \) denote the vector field defining the closed-loop system’s response, i.e.,

\[
\begin{bmatrix}
\dot{\Theta} \\
\dot{\omega}
\end{bmatrix} = X(\Theta, \omega).
\]
Then we take as output vector the tracking error in the angular momentum
\[ Y(t) = \Pi(t) = J\omega \in \mathbb{R}^3. \] (14)

The Lie derivative of a component \( Y_i \) of the output vector along the closed-loop vector field \( X \) is denoted by \( L_X Y_i \). The observability co-distribution is defined as
\[ d\mathcal{O}(\Theta, \omega) = \text{span}\{dL_X^k Y_i(\Theta, \omega), i = 1, 2, 3, k = 0, 1, 2, \ldots \}, \]
where \( L_X^0 Y_i = Y_i, L_X^2 Y_i = L_X(L_X Y_i) \) and so on. According to Corollary 2.3.5 of Isidori,\(^{10}\) the closed-loop system with the output function (14) is observable at a point \((\Theta, \omega) \in \mathbb{R}^6\) if the dimension of \( d\mathcal{O}(\Theta, \omega) \) is six.

We evaluate the Lie derivatives of the \( Y_i \) at the equilibrium \((I, 0) \in \text{TSO}(3)\) or of the closed-loop dynamics, where the disturbance moment \( M_d = 0 \) as \( \omega = 0 \). This corresponds to \((\Theta, \omega) = (0, 0)\) in the exponential coordinates. Computation of the first few vector fields in the observability co-distribution evaluated at \((\Theta, \omega) = (0, 0)\) confirm that its dimension is 6. Therefore, the system is locally observable at \((\Theta, \omega) = (0, 0)\), which corresponds to the equilibrium \((I, 0) \in \text{TSO}(3)\). This means that there exists a neighborhood \( \mathcal{N} \) of \((I, 0)\) such that the outputs \( Y_i(t) = 0 \Leftrightarrow \omega_i(t) = 0, t \geq 0, \) if and only if the state is \((I, 0)\). Hence, the equilibrium \((I, 0) \in \text{TSO}(3)\) is locally asymptotically stable.

From Lemma 1, we see that the other three equilibria of the closed-loop system (7) and (11) are given by \((Q_e, 0)\) such that \( Q_e \in E_e \setminus \{I\} \). The symmetric “Hessian” matrices (given by \( S = KQ \) in the proof of Lemma 1) have at least one negative eigenvalue, from which it follows that the corresponding linearized system is unstable at these equilibria. Thus, these three equilibria for the nonlinear closed-loop system are unstable. Following arguments similar in nature to those presented in Chaturvedi and McClamroch,\(^{6}\) it can be shown that each of these three equilibria have nontrivial unstable manifolds whose dimension is less than 6. Therefore, the set of all initial conditions that converge to these three unstable equilibria form a lower dimensional manifold in TSO(3) whose dimension is less than six.
Substituting \( \tau \) from (11) into the expression (17), we get
\[
\dot{V}(Q, \omega) = -\omega^T L\omega + \omega^T M_d - \sigma \omega^T u. \tag{18}
\]
Evaluating the last part of the right-hand side of (18), we have
\[
\omega^T M_d - \sigma \omega^T u = \omega^T M_d - \omega^T \left( \frac{\sigma \omega}{\|\omega\|} \right) = \omega^T M_d - \sigma \|\omega\| < 0
\]
when \( \|\omega\| > 0 \), from equation (10). For \( \omega = 0 \), \( \dot{V}(\cdot) \) is clearly zero. Therefore, \( \dot{V}(Q, \omega) \) is negative semidefinite along trajectories of the closed-loop system on TSO(3).

Recall that \( \Phi(\cdot) \) is a strictly increasing monotone function and SO(3) is compact. Hence, for any \((Q(0), \omega(0)) \in \text{TSO}(3)\), the set
\[
\mathcal{I} = \{(Q, \omega) \in \text{TSO}(3) : V(Q, \omega) \leq V(Q(0), \omega(0))\}
\]
is a compact, invariant set of the closed-loop system.6

By LaSalle’s invariant set theorem, it follows that all solutions that begin in \( \mathcal{I} \) converge to the largest invariant subset of \( V^{-1}(0) \) contained in \( \mathcal{I} \). From (18), we see that \( \dot{V}(Q, \omega) \equiv 0 \) implies \( \omega \equiv 0 \). Note that in this set we have both \( M_d = 0 \) and \( u = s = 0 \) since \( \omega \equiv 0 \). Substituting this into the closed-loop system equations, it can be shown that
\[
\dot{V}^{-1}(0) = \{(Q, \omega) \in \text{TSO}(3) : \omega \equiv 0, k_1 e_1^T Q^T e_1 + k_2 e_2^T Q^T e_2 + k_3 e_3^T Q^T e_3 \equiv 0\}
\]
In this case, each of the four points given by (15) are an equilibrium of the closed-loop dynamics in TSO(3). Therefore, by LaSalle’s theorem, all solutions of the closed-loop system converge to one of the equilibria in \( \mathcal{E} \cap \mathcal{I} \), where \( \mathcal{E} \) is given by (15).

From Lemma 2, we see that the only stable equilibrium in this equilibrium set is \((Q, \omega) = (I, 0)\). In fact as stated in Lemma 2, all solutions that converge to these three equilibria form a lower dimensional manifold. Thus, this set of unstable solutions is open and dense and has measure zero in \text{TSO}(3) (see also the global treatment in Chaturvedi and McClamroch6). Solutions of the closed-loop system that do not start in this manifold, thus converge asymptotically to the stable equilibrium \((Q, \omega) = (I, 0)\). Therefore, the domain of attraction of this equilibrium is almost global.

Since this is also the desired equilibrium for asymptotic tracking, the closed-loop system performs almost global asymptotic tracking in this case. The more general case where \( M_d \) does not vanish as \( \omega \to 0 \), is also more involved. However, we can show that the closed-loop system is at least locally practically asymptotically stable at the desired point \((Q, \omega) = (I, 0)\) for trajectory tracking when \( M_d \) has a small magnitude.

V. Simulation Results

In this section we obtain simulation results for a model of a spacecraft in a circular Keplerian orbit with an orbital rate of 0.0012 rad/sec. The moment of inertia of the spacecraft is \( J = \text{diag}(10, 15, 7.5) \) kg-m\(^2\).

The desired trajectory is given by
\[
R_d(0) = I, \quad \Omega_d(t) = 5 \sin(0.1t) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ deg/sec.}
\]
The initial attitude and angular velocity of the spacecraft are taken to be:
\[
R(0) = \begin{bmatrix} 0.2065 & 0.8760 & -0.4359 \\ -0.9733 & 0.2294 & 0 \\ 0.1000 & 0.4243 & 0.9000 \end{bmatrix}, \quad \Omega(0) = \begin{bmatrix} 10 \\ 2 \\ 3 \end{bmatrix} \text{ deg/sec.}
\]
A disturbance input torque modeled by a band limited white noise is applied to this system, such that its norm is always less than 0.06 Nm. Therefore, we choose the norm bound for $\sigma = 0.061$ Nm. The controller gain matrix for angular velocity damping is $L = 2\text{diag}(\sigma + 0.01, \sigma + 0.005, \sigma + 0.001)$. Finally, the control function $\Phi(x) = 0.5x$. This defines all the control parameters used in this simulation.

The simulation results are plotted in two sets of four plots each. In the first set of results presented (figures 1 and 2), we use the control law (11) with the disturbance compensation part. The second set of results (figures 3 and 4) are obtained for the closed-loop system with the control law with only trajectory tracking terms, but without the disturbance compensation. Comparing pairwise the numerical simulation results in figures 1 and 3, we see that the angular velocity and attitude converge to the desired trajectory with the disturbance rejection controller, but not without it. The control torque magnitudes for the disturbance rejection controller in Figure 2 is comparable to the control magnitudes obtained without the disturbance rejection in Figure 4, both being less than 2.5 Nm, although the disturbance rejection controller shows an overall decreasing trend in control torque magnitude. This effectively demonstrates the utility of the disturbance rejection controller in the presence of such bounded disturbances.

![Figure 1](image1.png)  
![Figure 2](image2.png)  

**Figure 1.** Angular velocity (left) and attitude error measure (right) for controller with disturbance compensation.

**Figure 2.** Control torque magnitude (left) and disturbance moment components (right) acting on the spacecraft for controller with disturbance compensation.
VI. Conclusions

In this paper, we presented an attitude and angular velocity tracking control logic for a rigid body in an attitude-dependent potential. This control law is \textit{continuous in the attitude and angular velocity states}, and is therefore implementable using actuators that produce continuous torque profiles like reaction wheels and magnetic torquerods. This control law achieved \textit{almost global asymptotic tracking for a rigid body system even in the presence of disturbance torques that vanish when the angular velocity tracking error vanishes}. Analytical and numerical results show that this controller achieves disturbance rejection for this class of disturbance torques, with accurate trajectory tracking. Numerical simulation results also show that feedback control without the disturbance rejection part fails to converge to the desired trajectory in the presence of such disturbance inputs. The future goal of this research would be to extend such tracking control schemes with good global properties to disturbance rejection of bounded but persistent disturbances, taking into account explicit bounds on the torques produced by the actuators used for control.

References


